Wake-Induced Full-Span Instability of Bundle Conductor Transmission Lines

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Linearized equations of motion are used to determine the boundaries of instability for wake-induced full-span oscillation of a bundle conductor transmission line. For normal conductors, it is found that instability occurs over a wide range of bundle tilt angles at most wind speeds. As the vertical natural frequency of the bundle is increased, additional instabilities are produced at zero tilt. However, increasing the twisting natural frequency tends to stabilize the bundle.

Introduction

AS voltage levels in overhead electrical transmission lines have increased, it has been necessary to go from single conductor lines to multiple or bundle conductors. Bundles of two, three, and four conductors are already in use, and six conductor bundles now are being considered.

It has been found that winds acting on these bundle conductors produce vibration problems different from those on single conductors. In addition to the galloping and aeolian vibration experienced by single conductors, bundle conductors also are susceptible to a low-frequency oscillation caused by wake interference effects between the conductors in the bundle. The oscillations are of two types: subspan oscillation in which the conductors oscillate between the spacers in the bundle, and full-span oscillation in which the bundle vibrates as a whole between the support towers. Subspan oscillation has been studied extensively[1-4] and is fairly well understood. However, very little consideration has been given to the full-span motion. Simpson and Lawson[5] have investigated full-span oscillation with particular emphasis on the effect of the presence of ice on the conductors.

In this paper, linearized equations of motion for a conductor bundle are used to find stability boundaries for oscillation of the bundle under the action of wake-induced aerodynamic forces.

Equations of Motion

For the purposes of this study, consideration has been limited to two-conductor bundles. This restriction is made for several reasons. First, two-conductor bundles are the most common; thus there are more aerodynamic data available for the two-conductor configuration. Most subspan oscillation studies also have been limited to the two-conductor bundle. Moreover, in full-span oscillation the main differences between bundles of two, three, or more conductors will be in the values of the various aerodynamic terms. Therefore, the equations of motion for full-span oscillation of any bundle will be basically the same.

The bundle then is assumed to consist of two taut wires connected by rigid, massless spacers. A coordinate system is chosen with the x axis horizontal and normal to the bundle, the y axis along the centerline of the bundle, and the z axis vertical (Fig. 1). It is assumed that the sag of the bundle is small compared to the span and that oscillations of the bundle involve only small displacements away from the no-wind equilibrium position. With these assumptions, there is no mechanical coupling between the three degrees of freedom (the degrees of freedom being displacement in the x and z directions and rotation about the y axis).

The basic equations of motion for the bundle are

$$m\frac{\partial^2 x}{\partial t^2} - \frac{\partial}{\partial y}\left[2T\frac{\partial x}{\partial y}\right] = F_x(x,z,\theta,\dot{x},\dot{z},\dot{\theta},V) \tag{1a}$$

$$m\frac{\partial^2 z}{\partial t^2} - \frac{\partial}{\partial y}\left[2T\frac{\partial z}{\partial y}\right] = mg + F_z(x,z,\theta,\dot{x},\dot{z},\dot{\theta},V) \tag{1b}$$

$$\frac{1}{4}md^2\frac{\partial^2 \theta}{\partial t^2} - \frac{\partial}{\partial y}\left[\frac{1}{2}d^2T\frac{\partial \theta}{\partial y}\right] = M_y(x,z,\theta,\dot{x},\dot{z},\dot{\theta},V) \tag{1c}$$

where m is the mass of the bundle per unit length, T is the tension in the bundle, and F_x, F_z, and M_y are the aerodynamic force and moment per unit length acting on the bundle. The aerodynamic force and moment will, in general, be functions of the position and velocity of the bundle relative to the oncoming wind. For small displacements, the moment of inertia of the bundle ($\frac{1}{4}md^2$) is a constant.

Let

$$x = x_0(t)f_x(y), \quad z = z_0(t)f_z(y), \quad \theta = \theta_0(t)f_\theta(y) \tag{2}$$

then

$$m\ddot{x}_0 f_x - (2Tx_0 f_x')' = F_x \tag{3a}$$

$$m\ddot{z}_0 f_z - (2Tz_0 f_z')' = mg + F_z \tag{3b}$$

$$\tfrac{1}{4}md^2\ddot{\theta}_0 f_\theta - (\tfrac{1}{2}md^2 T\theta_0 f_\theta')' = M_y \tag{3c}$$

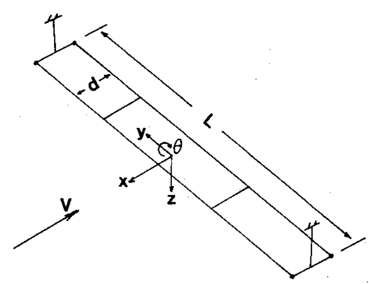

Fig. 1 Bundle geometry.

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where dots denote differentiation with respect to time and primes denote differentiation with respect to y.

Since the bundle is supported at the ends, the spatial variation of the displacement must be of the type

$$f_x = f_z = f_\theta = \sin\,(n\pi y/L) \tag{4}$$

Substituting into Eq. (3) and integrating over the span produces the equations

$$\ddot{x}_0 + 2(n\pi/L)^2(T/m)x_0 = (1/m)F_x(x_0, z_0, \theta_0, \dot{x}_0, \dot{z}_0, \dot{\theta}_0, V) \tag{5a}$$

$$\ddot{z}_0 + 2(n\pi/L)^2(T/m)z_0 = (\pi/2)g + (1/m)F_z(x_0, z_0, \theta_0 \dot{x}_0, \dot{z}_0, \dot{\theta}_0, V) \tag{5b}$$

$$\ddot{\theta} + 2(n\pi/L)^2(T/m)\theta_0 = (4/md^2)M_y(x_0, z_0, \theta_0, \dot{x}_0, \dot{z}_0, \dot{\theta}_0, V) \tag{5c}$$

The coefficients of the second-terms in Eq. (5) usually are associated with the square of the structural natural frequency. Thus the equations may be written as

$$\ddot{x}_0 + \omega_x^2 x_0 = F_x/m \tag{6a}$$

$$\ddot{z}_0 + \omega_z^2 z_0 = (\pi/2)g + (F_z/m) \tag{6b}$$

$$\ddot{\theta}_0 + \omega_\theta^2 \theta_0 = 4M_y/md^2 \tag{6c}$$

Even though the derivation just used gives equal values for the natural frequencies, there is no reason why these equations could not be applied also when $\omega_x \neq \omega_z \neq \omega_\theta$. In fact, in a true catenary, the vertical natural frequency is slightly higher than the horizontal frequency. It also has been shown[1] that subspan oscillation is possible only if $\omega_z \neq \omega_x$. The natural frequencies in (6) thus are assumed to be given, independent quantities representing known structural properties of the bundle.

The aerodynamic load on the bundle will consist of a lift, a drag, and a pitching moment (Fig. 2). These aerodynamic loads arise from the wake interaction between the conductors in the bundle and will be functions of the angle between the bundle and the oncoming wind. Thus the aerodynamic loads are functions of the total angle of attack Λ, where

$$\Lambda = \theta + \alpha \tag{7}$$

The relative wind angle α is given by

$$\alpha = \tan^{-1}[\dot{z}_0/(V + \dot{x}_0)] \tag{8}$$

which is

$$\alpha = \dot{z}_0/V \tag{9}$$

for small disturbances.

The aerodynamic loads are given in terms of coefficients

$$L_f = \tfrac{1}{2}\rho V_c^2 c C_L \tag{10a}$$

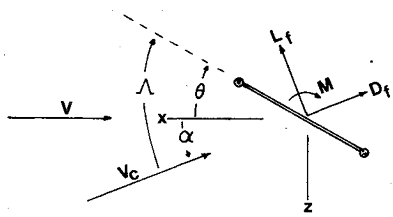

Fig. 2 Aerodynamic load on bundle.

$$D_f = \tfrac{1}{2}\rho V_c^2 c C_D \tag{10b}$$

$$M = \tfrac{1}{2}\rho V_c^2 d c C_M \tag{10c}$$

where

$$V_c = [(V + \dot{x}_0)^2 + \dot{z}_0{}^2]^{1/2} = [V^2 + 2V\dot{x}_0]^{1/2} \tag{11}$$

is the total velocity relative to the wind. L_f is the lift force normal to the wind, and D_f is the drag force along the wind. c is the diameter of each of the conductors in the bundle, and ρ is the density of the atmosphere.

For small angles, the forces on the bundle are therefore

$$F_x = \tfrac{1}{2}\rho V_c^2 c(C_L\alpha - C_D) \tag{12a}$$

$$F_z = -\tfrac{1}{2}\rho V_c^2 c(C_L + C_D\alpha) \tag{12b}$$

$$M_y = \tfrac{1}{2}\rho V_c^2 d c C_M \tag{12c}$$

The aerodynamic coefficients depend only on Λ and may be written as

$$C_L = C_{L_0} + C_{L_\alpha}\Lambda \tag{13a}$$

$$C_D = C_{D_0} + C_{D_\alpha}\Lambda \tag{13b}$$

$$C_M = C_{M_0} + C_{M_\alpha}\Lambda \tag{13c}$$

by means of a Taylor Series expansion about the no-wind angle. The aerodynamic forces then are

$$F_x = qc[-C_{D_0} - 2C_{D_0}(\dot{x}_0/V) + (C_{L_0} - C_{D_\alpha})(\dot{z}_0/V) - C_{D_\alpha}\theta_0] \tag{14a}$$

$$F_z = qc[C_{L_0} + 2C_{L_0}(\dot{x}_0/V) + (C_{D_0} + C_{L_\alpha})(\dot{z}_0/V) + C_{L_\alpha}\theta_0] \tag{14b}$$

$$M_y = qdc[C_{M_0} + 2C_{M_0}(\dot{x}_0/V) + C_{M_\alpha}(\dot{z}_0/V) + C_{M_\alpha}\theta_0] \tag{14c}$$

where

$$q = \tfrac{1}{2}\rho V^2 \tag{15}$$

For a two-conductor bundle, all of the aerodynamic coefficients in (14) can be extracted from the data given by Wardlaw and Cooper.[6]

With these aerodynamic forces, the equations of motion become

$$\ddot{x}_0 + 2(qc/mV)C_{D_0}\dot{x}_0 + \omega_x^2 x_0 - (qc/mV)(C_{L_0} - C_{D_\alpha})\dot{z}_0 + (qc/m)C_{D_\alpha}\theta_0 = -(qc/m)C_{D_0} \tag{16a}$$

$$\ddot{z}_0 + (qc/mV)(C_{D_0} + C_{L_\alpha})\dot{z}_0 + \omega_z^2 z_0 + 2(qc/mV)C_{L_0}\dot{x}_0 + (qc/m)C_{L_\alpha}\theta_0 = (\pi/2)g - (qc/m)C_{L_0} \tag{16b}$$

$$\ddot{\theta}_0 + [\omega_\theta^2 - 4(qc/md)C_{M_\alpha}]\theta_0 - 8(qc/mdV)C_{M_0}\dot{x}_0 - 4(qc/mdV)\dot{z}_0 = 4(qc/md)C_{M_0} \tag{16c}$$

It is convenient to nondimensionalize the equations. Let

$$\hat{x} = x_0/d, \quad \hat{z} = z_0/d, \quad \hat{\theta} = \theta_0$$

$$\hat{t} = t/t^*, \quad \hat{\omega} = t^*\omega \tag{17}$$

where

$$t^* = d/V \tag{18}$$

Defining a relative mass ratio as

$$\mu = \rho dc/2m \tag{19}$$

and using D to denote a differentiation with respect to $\hat{t}(D = d/d\hat{t})$, the equations of motion for the bundle may be written in matrix form as

$$\begin{bmatrix} D^2+2\mu C_{D_0}D+\hat{\omega}_x^2 & -\mu(C_{L_0}-C_{D_\alpha})D & \mu C_{D_\alpha} \\ 2\mu C_{L_0}D & D^2+\mu(C_{D_0}+C_{L_\alpha})D+\hat{\omega}_z^2 & \mu C_{L_\alpha} \\ -8\mu C_{M_0}D & -4\mu C_{M_\alpha}D & D^2+(\hat{\omega}_\theta^2-4\mu C_{M_\alpha}) \end{bmatrix} \begin{Bmatrix} \hat{x} \\ \hat{z} \\ \hat{\theta} \end{Bmatrix} = \begin{Bmatrix} -\mu C_{D_0} \\ (\pi/2)(dg/V^2)-\mu C_{L_0} \\ 4\mu C_{M_0} \end{Bmatrix} \tag{20}$$

With the equations in this form, the wind velocity appears only in the parameters $\hat{\omega}_x$, $\hat{\omega}_z$, $\hat{\omega}_\theta$. The physical properties of the bundle are included in μ and in the $\hat{\omega}$'s and the no-wind position of the bundle is taken care of by C_{L_0}, C_{D_0}, and C_{M_0}.

Bundle Instability

Continued oscillation of the bundle will be possible only when the parameters are such that the system has neutral dynamic stability or is dynamically unstable. This will occur whenever the characteristic equation of the system has a root with a zero or positive real part. Thus, the instability boundaries for the system can be obtained from the roots of the characteristic equation.

In order to evaluate the stability, a representative bundle was assumed to consist of two smooth conductors with a 10-diameter separation ($d/c = 10$). Typical natural frequencies for this bundle are, in radians per second,

$$\omega_x = 1.110, \quad \omega_z = 1.221, \quad \omega_\theta = 1.528 \tag{21}$$

The aerodynamic properties of the bundle are obtained by combining the force data for each conductor as given by wind-tunnel tests.[6] Figures 3-8 show the bundle aerodynamic coefficients for negative Λ. The drag coefficient is symmetric with respect to Λ, but the lift and moment coefficients are antisymmetric.

Variables that will affect the stability of the bundle are the wind speed and the no-wind tilt angle. The no-wind tilt angle is the angle, relative to the horizontal, at which the bundle is hung. This angle is normally zero, but it can be varied by changing the support hardware. The effect of wind speed and tilt angle on the stability of the bundle is shown in Fig. 9.

For low wind speeds, the bundle is stable over most tilt angles. However as wind speed increases, instability occurs over a wide range of tilt angles. At high wind speeds, the bundle is stable only for very small or very large tilt angles. There are no wake interference effects for tilt angles above 25°, so that the bundle acts as a single cable in this region.

Sag of the bundle causes the vertical natural frequency (ω_z) of the bundle to be higher than the horizontal natural frequency (ω_x). For small sag the difference is slight, but as sag increases the vertical natural frequency becomes much higher. Theoretical calculations[7,8] have shown that the vertical frequency in the first mode may be as much as 2.5 times the corresponding horizontal frequency for sag of the order of one-eighth the span. The effect of vertical natural frequency on the stability of the bundle is shown in Figs. 10-13.

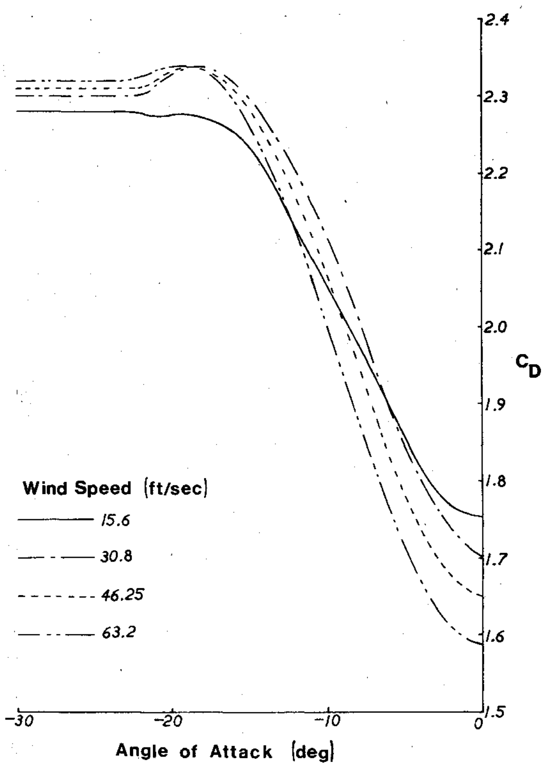

Fig. 3 Drag coefficient.

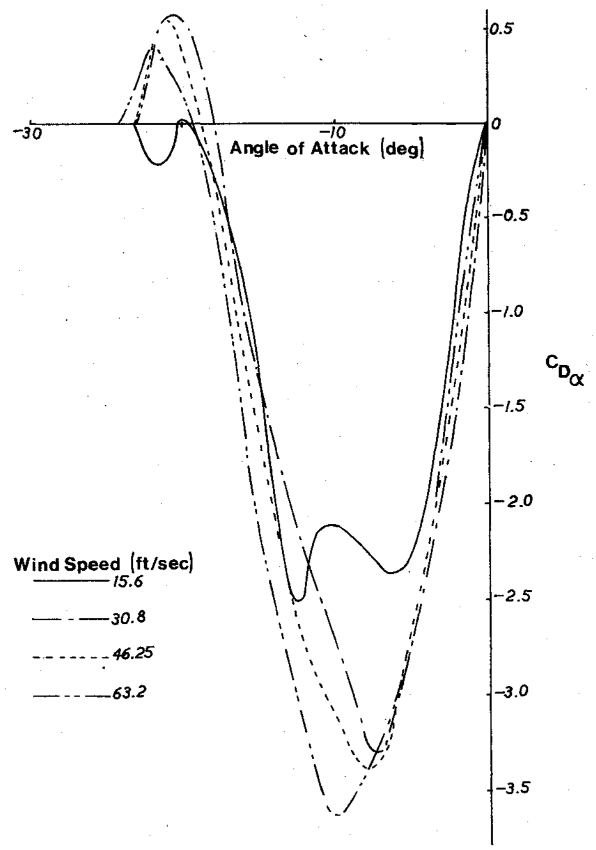

Fig. 4 Drag slope.

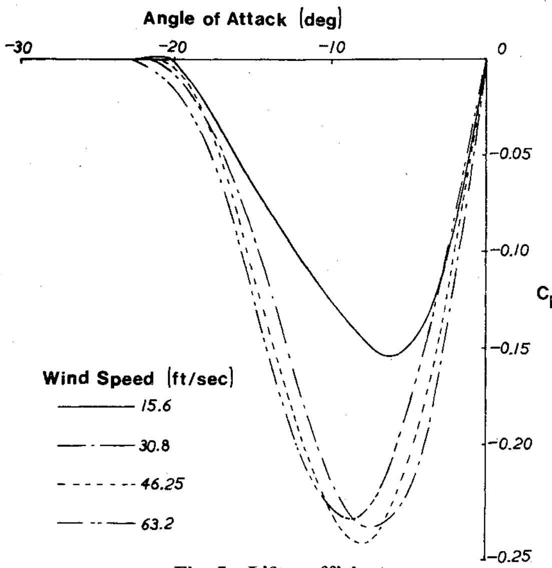

Fig. 5 Lift coefficient.

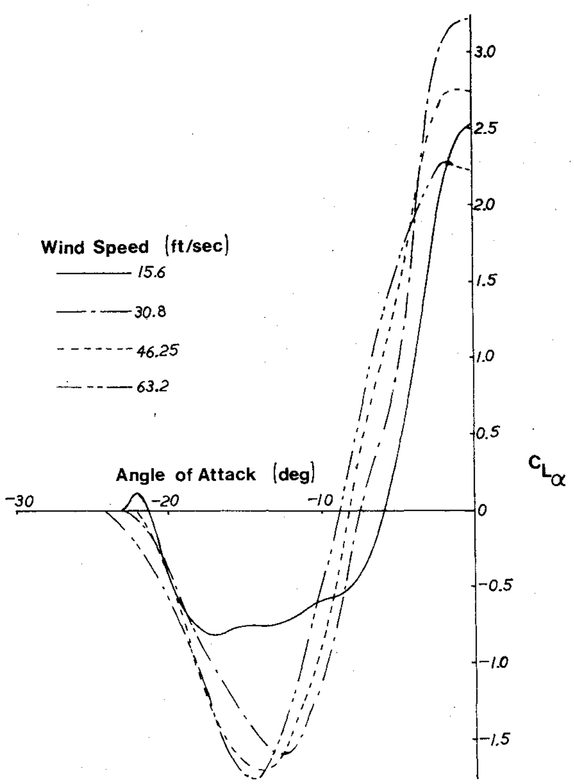

Fig. 6 Lift slope.

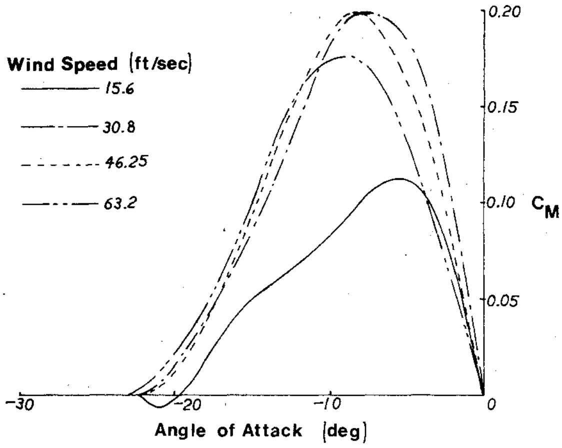

Fig. 7 Moment coefficient.

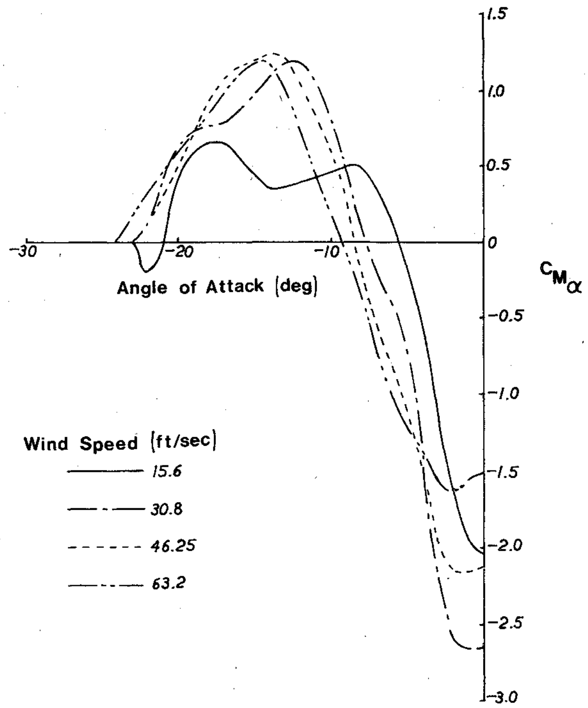

Fig. 8 Moment slope.

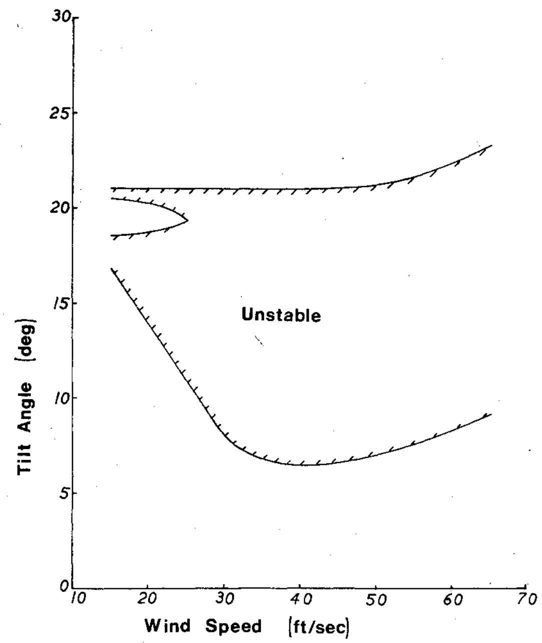

Fig. 9 Bundle stability boundary.

High vertical frequencies tend to increase the size of the unstable region in tilt angle. As ω_z is increased, a new unstable region occurs for small tilt angles. This instability, however, disappears for high wind speeds. Thus, for bundles with large sag there are very few tilt angles for which the bundle will be stable at all wind speeds.

Increasing the twisting or torsional natural frequency of the bundle tends to have a stabilizing effect on the motion. At low wind speeds (Figs. 14 and 15), the size of the unstable region in tilt angle is reduced greatly. The effect of twisting frequency is not as pronounced at the higher speeds (Figs. 16 and 17), but an improvement in stability still is obtained.

The wind speed has a large influence on the time to double of the unstable motion. For low wind speeds with the nominal

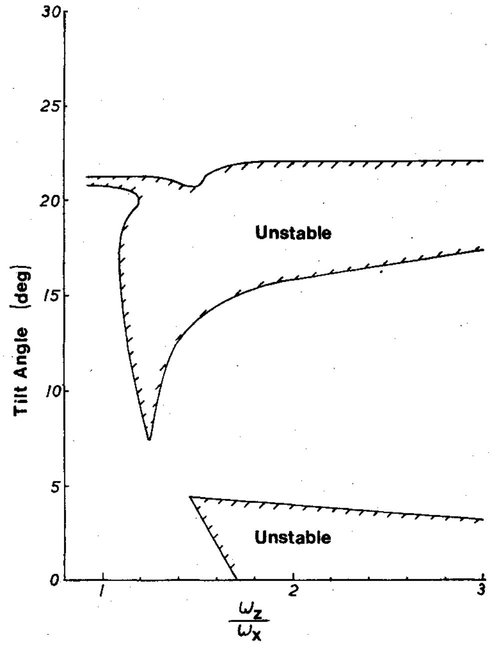

Fig. 10 Effect of vertical frequency; $V = 15.6$ fps.

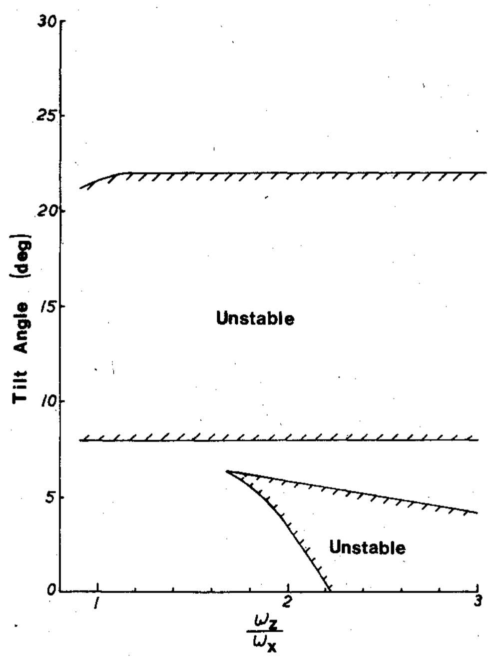

Fig. 11 Effect of vertical frequency; $V = 30.8$ fps.

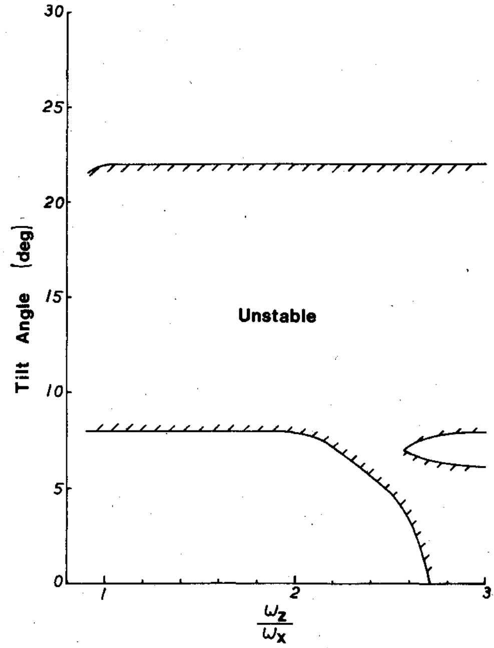

Fig. 12 Effect of vertical frequency; $V = 46.25$ fps.

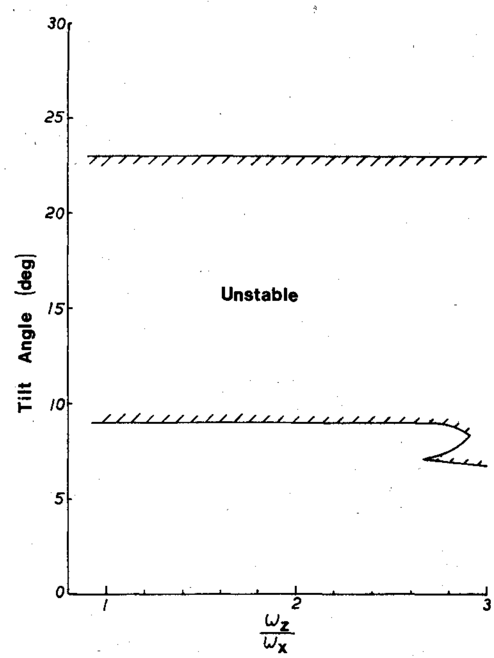

Fig. 13 Effect of vertical frequency; $V = 63.2$ fps.

bundle, the time to double for the instability is on the order of 1000 sec. At high wind speeds, however, the time to double is reduced to about 0.1 sec. Increases in the vertical natural frequency also dramatically reduce the time to double.

Conclusions

Calculations made with a linearized model have shown that the full-span oscillation of a two-conductor bundle transmission line will be unstable over a wide range of bundle tilt angles and wind speeds. Increasing the vertical natural frequency of the bundle introduces further instability near zero tilt. The size of the unstable region in tilt angle can be reduced, however, by increasing the twisting natural frequency of the bundle.

It has been proposed that the tilt angle of the bundle be increased as a means of reducing subspan oscillation. The results obtained here, however, indicate that such a move could result in full-span instability.

References

[1] Simpson, A., "Stability of Subconductors of Smooth Circular Cross-Section," *Proceedings of the Institution of Electrical Engineers*, Vol. 117, April 1970, pp. 741-750.

[2] Simpson, A., "Wake Induced Flutter of Circular Cylinders: Mechanical Aspects," *Aeronautical Quarterly*, Vol. 22, May 1971, pp. 101-118.

[3] Ko, R. G. and Wardlaw, R. L., "Three-Dimensional Analysis on the Wind Induced Subspan Oscillations of Bundled Conductors," Paper C 74 060-0, Jan. 1974, Institute of Electrical and Electronics Engineers.

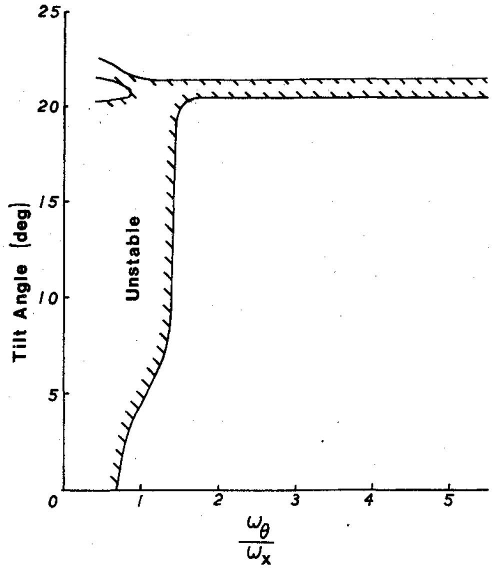

Fig. 14 Effect of torsional frequency; $V = 15.6$ fps.

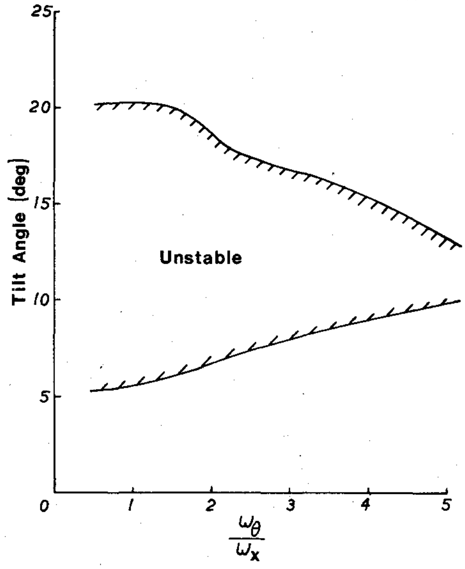

Fig. 16 Effect of torsional frequency; $V = 46.25$ fps.

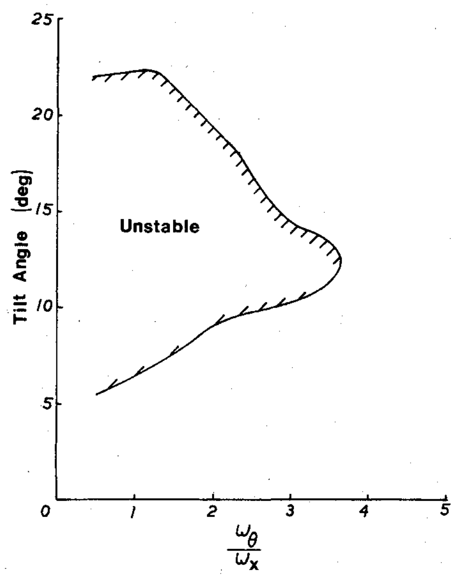

Fig. 15 Effect of torsional frequency; $V = 30.8$ fps.

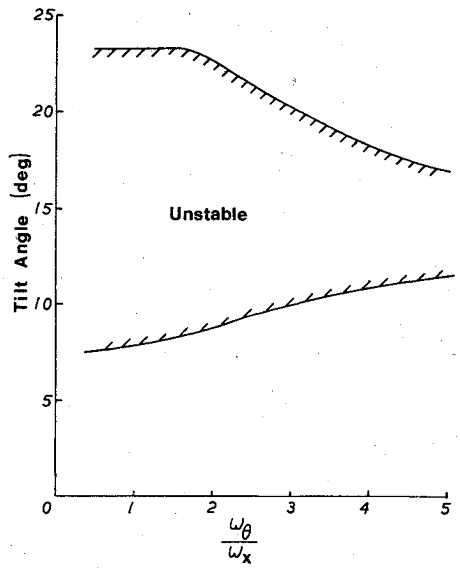

Fig. 17 Effect of torsional frequency; $V = 63.2$ fps.

[4]Rawlins, C. B., "Effect of Wind Turbulence in Wake-Induced Oscillations of Bundled Conductors," Paper C 74 444-6, July 1974, Institute of Electrical and Electronics Engineers.

[5]Simpson, A. and Lawson, T. V., "Oscillations of 'Twin' Power Transmission Lines," *Proceedings of the Symposium on Wind Effects on Buidlings and Structures*, April 1968, Loughborough University.

[6]Wardlaw, R. L. and Cooper, K. R., "A Wind Tunnel Investigation of the Steady Aerodynamic Forces on Smooth and Stranded Twin Bundled Power Conductors for the Aluminum Company of America," LTR-LA-117, Aug. 1973, National Aeronautical Establishment, Ottawa, Canada.

[7]Soler, A. I., "Dynamic Response of Single Cables with Initial Sag," *Journal of the Franklin Institute*, Vol. 290, Oct. 1970, pp. 377-387.

[8]Irvine, H. M. and Caughey, T. K., "The Linear Theory of Free Vibrations of a Suspended Cable," *Proceedings of the Royal Society (London)*, Vol. A 341, Dec. 1974, pp. 299-315.